

The Research of PID algorithm On Attitude controller for Four-rotor UAV

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Abstract: In order to solve the flight control problem of Four-rotor UAV. The transfer function of the pitch, roll and yaw passages is established by the coordinate transformation and the attitude calculation of the four-rotor UAV. The PID controller is used to control the flight attitude of the four-rotor unmanned aerial vehicle. MATLAB / Simulink simulation results show that: PID controller can effectively achieve the quad-rotor UAV control, significantly reduced the steady-state error of UAS such that the stability of the entire UAV system has been improved. It has certain reference value and guidance significance to the research of UAV system stability in the future.

Key words: Four-rotor UAV; Attitude control; PID controller; MATLAB / Simulink simulation

I. INTRODUCTION

With the continuous development of space technology, quad-rotor UAV with its unique shape and lightweight structure, simple and can be fixed VTOL hover, and operation of flexible control, etc., are concerned by more and more scholars and researchers^[1].

Four-rotor unmanned aerial vehicle is different from the helicopter. It changes the flight attitude by changing the speed of the motor mounted at the four end points of the fuselage without the need for a tail^[2]. So the power of quad-rotor UAV stronger, more stable flight attitude.

In recent years, the application of four-rotor unmanned aerial vehicles more and more widely. So ensuring the quality of flight has become a hot research. The key factor to determine the quality of the flight is the control effect of the four-rotor flight attitude.

At present, the attitude modeling of four-rotor unmanned aerial vehicle (UAV) is basically belong to the category of nonlinear control^[3]. Non-linear control requires a high-precision control model. So under the premise of a certain error, using PID control algorithm to control the effect is more obvious. In this paper, a PID controller is designed for this characteristic, which makes the performance of the system more good.

II. NONLINEAR MATHEMATICAL MODEL OF FOUR ROTOR

Quad-rotor UAV having the structure X-type, Which is a non-linear input 4 and 6 degrees of freedom, multi-variable, typically due to high coupling drive system^[4,5]. In order to make the dynamic model of four-rotor unmanned aerial vehicle (UAV) not general, the paper puts forward some reasonable hypotheses:

- ① the quad-rotor UAV as rigid, symmetrical structure and quality. So the inertial matrix can be defined as a diagonal matrix I .
- ② Four-rotor UAV center of gravity, center of mass and geometric center is consistent with the origin UAVs built for double coordinate system.
- ③ Ignore the impact of quad-rotor UAV suffered surface air resistance and other factors.
- ④ Quad-rotor UAV 4 lift was positively correlated with the square of the corresponding motor speed.

Quad-rotor UAV structural model^[6] shown in Figure 1-1:

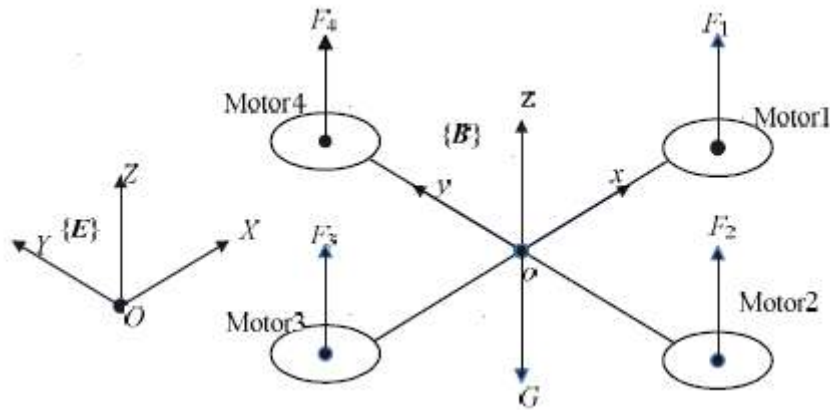


Figure 1-1 Quad-rotor UAV structural model

In order to facilitate the establishment of quad-rotor UAV model, we need to establish two coordinate systems, Inertial coordinate system E (OXYZ) and quad-rotor UAV body coordinate system B (oxyz). The inertial coordinate system E is characterized by Euler angle. Euler angles are used to determine a set of independent variables fixed rigid body rotating position. Here, θ represents the pitch angle, ϕ represents the roll angle, ψ represents the yaw angle. According to the angular momentum theorem, the momentum moment of the moment to the rigid body is equal to the change of the rigid body angular momentum.

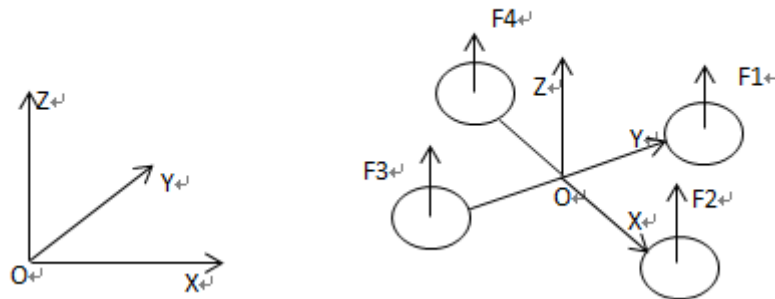


Figure 1-2 inertial coordinate system

According to Newton's second law, $F = m \frac{dv}{dt}$, F is a quad-rotor UAV by the sum of external forces. m is the

quality of the four-rotor unmanned aerial vehicle. v is the speed of four-rotor unmanned aerial vehicle. F is decomposed into four rotary wing unmanned aerial vehicle body coordinate system, three components can be obtained. Respectively F_x 、 F_y 、 F_z . The corresponding angular velocity ω decomposition to the four-rotor

UAV body coordinate system can get three components. Respectively, p 、 q 、 r .

According to the four-rotor UAV dynamic equation^[7], $M = \frac{dH}{dt}$ M is the sum of the moments of the four-rotor UAV. H is the absolute moment of momentum of the four-rotor UAV relative to the ground coordinate system.

In summary, according to the four-rotor unmanned aerial vehicle of the force analysis, Newton's second law, and four-rotor UAV dynamic equation. We can get the four-rotor unmanned aerial vehicle linear motion equation:

$$\left\{ \begin{aligned} \ddot{x} &= \frac{F_x - K_1 \dot{x}}{m} = \frac{k_t \sum_{i=1}^4 \omega_i^2 (\cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi) - K_1 \dot{x}}{m} \\ \ddot{y} &= \frac{F_y - K_2 \dot{y}}{m} = \frac{k_t \sum_{i=1}^4 \omega_i^2 (\sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi) - K_2 \dot{y}}{m} \\ \ddot{z} &= \frac{F_z - K_3 \dot{z} - mg}{m} = \frac{k_t \sum_{i=1}^4 \omega_i^2 (\cos \theta \cos \phi) - K_3 \dot{z}}{m} - g \end{aligned} \right. \quad (1-1)$$

According to the calculation and analysis of moment of momentum, M can be decomposed into the quad-rotor UAV airframe coordinates. Respectively, M_x 、 M_y 、 M_z .

In summary, combined with Conversion Model Euler angles and the angular velocity between the quad-rotor UAV and the Inertial diagonal matrix I .We can get four-rotor unmanned aerial vehicle angular motion equation:

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{M_x + (I_x - I_z)qr}{I_x} \\ \frac{M_y + (I_z - I_x)rp}{I_y} \\ \frac{M_z + (I_x - I_y)pq}{I_z} \end{bmatrix} \quad (1-2)$$

There are four groups of four-rotor UAV control channel and these four groups of channels are independent of each other. Assuming control inputs are U_1 、 U_2 、 U_3 、 U_4 .

According to a four-rotor UAV attitude kind of change, we can obtain the following relationship:

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} F_1 + F_2 + F_3 + F_4 \\ F_4 - F_2 \\ F_3 - F_1 \\ F_2 + F_4 - F_1 - F_3 \end{bmatrix} = \begin{bmatrix} k_t \sum_{i=1}^4 \omega_i^2 \\ k_t (\omega_4^2 - \omega_2^2) \\ k_t (\omega_3^2 - \omega_1^2) \\ k_d (\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2) \end{bmatrix} \quad (1-3)$$

U_1 produces a vertical movement effect, which is the amount of vertical control, U_2 creates a rollover effect for the rollover control, U_3 produces the pitch motion effect as the pitch control amount, U_4 produces a yaw motion effect, which is the amount of yaw control, ω_i is the rotational speed of each rotor, F_i is the lift generated by each rotor.

In summary, combined with the linear motion equation and angular motion equation of the four - rotor unmanned aerial vehicle (UAV), a simplified mathematical model can be obtained under the condition of neglecting the resistance and the change of the surface factors:

$$\left\{ \begin{array}{l} \ddot{x} = \frac{(\cos \psi \sin \theta \cos \phi + \sin \psi \sin \theta)U_1}{m} \\ \ddot{y} = \frac{(\sin \psi \sin \theta \cos \phi - \cos \psi \sin \theta)U_1}{m} \\ \ddot{z} = \frac{(\cos \phi \cos \theta)U_1}{m} - g \\ \ddot{\psi} = \frac{[lU_2 + \dot{\theta}\dot{\psi}(I_y - I_z)]}{I_x} \\ \ddot{\theta} = \frac{[lU_3 + \dot{\phi}\dot{\psi}(I_z - I_x)]}{I_y} \\ \ddot{\phi} = \frac{[lU_4 + \dot{\phi}\dot{\theta}(I_x - I_y)]}{I_z} \end{array} \right. \quad (1-4)$$

In this, l represents the distance between the rotor center and the origin of the coordinate system. $\ddot{x}, \ddot{y}, \ddot{z}$ is a four-rotor displacement in the x, y, z -axis direction of the coordinate system. I_x, I_y, I_z is respectively the moment of inertia of the four rotors about x, y, z -axis

In order to simplify the system complexity, it is assumed that there exists a satisfied relationship between the attitude angle and the angular velocity, as follows:

$$\dot{\theta} = q, \dot{\phi} = p, \dot{\psi} = r.$$

Ignoring small perturbations premise, we can get four-rotor UAV state equation:

$$m\dot{x} = Ax + Bu$$

In this $x = [\dot{x}, \dot{y}, \dot{z}, p, q, r, \theta, \phi, \psi]^T$, $u = [u_1, u_2, u_3, u_4]$.

According to the performance parameters of the experimental prototype and the transfer function $G(s) = (sI - A)^{-1}B$, the transfer function of each channel can be obtained clearly.

Table 1 prototype performance parameters

parameter name	Value	unit
l	0.25	m
m	1.25	kg
k_t	$3.13 \cdot 10^{-5}$	$N \cdot s^2$

k_d	$7.5 \cdot 10^{-7}$	$N \cdot ms^2$
I_x	$2.353 \cdot 10^{-3}$	$kg \cdot m^2$
I_y	$2.353 \cdot 10^{-3}$	$kg \cdot m^2$
I_z	$5.262 \cdot 10^{-2}$	$kg \cdot m^2$

Table 2 the channel transfer function

Channel Name	Channel transfer function
Pitch channel	$G_1 = \frac{\theta}{u_1} = \frac{56.95s + 4391}{s^3 + 105s^2 + 870s + 4430}$
Roll channel	$G_2 = \frac{\phi}{u_2} = \frac{65s + 4560}{s^3 + 109s^2 + 1023s + 2935}$
Yaw channel	$G_3 = \frac{\psi}{u_3} = \frac{105}{s^2 + 413s}$
Z axis	$G_4 = \frac{z}{u_4} = \frac{\dot{z}}{su_4} = \frac{1.63}{s(s + 5)}$

III. PID CONTROLLER DESIGN

In the analog control system, the system consists of a PID controller and the controlled object, analog PID control system block diagram shown in Figure 2-1:

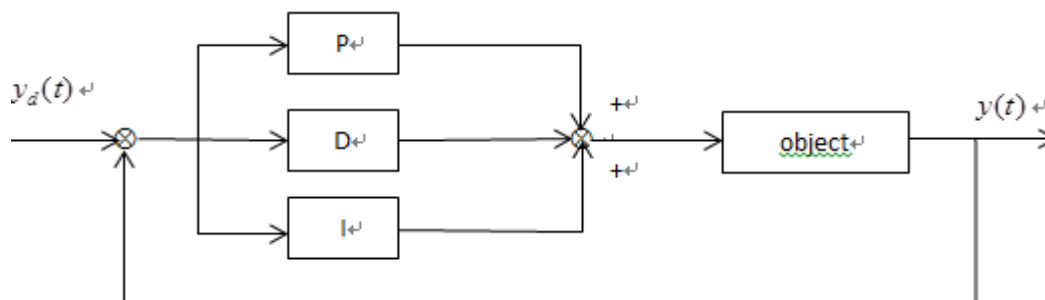


Figure 2-1 block diagram of analog PID control system

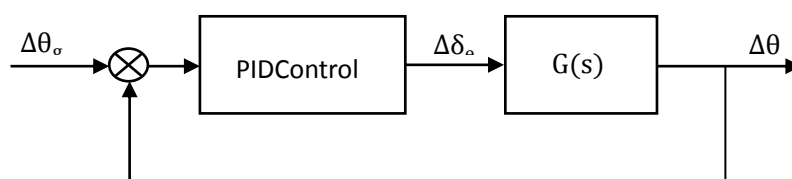


Figure 2-2 PID control system block diagram

According to the block diagram of PID control system, the simulation system is built on MATLAB / Simulink simulation platform, and the attitude step response curve of the control system is obtained finally. Control parameters in Table 3, the corresponding attitude diagram shown in Figure 2, Figure 3, Figure 4.

Table 3 Control parameter settings

Channel Name	k_p	k_i	k_d
Pitch angle	1	2	1
Roll angle	1.5	3.5	0.5
Yaw angle	8	8	0.5
Z-axis direction	8	10	0.8

IV. PID CONTROL OF MATLAB / SIMULINK SIMULATION

Based on the transfer function of each channel and the simplified model of the four-rotor UAV, a Simulink simulation model was set up to control each channel independently.

Taking the pitch channel as an example, the controller can be used to obtain the simulation result of other channel control.

For the pitch channel, the transfer function is:
$$G_1 = \frac{\theta}{u_1} = \frac{56.95s + 4391}{s^3 + 105s^2 + 870s + 4430}$$

Design the output of the PID controller is:

$$\Delta\delta_e = K_p e + K_i \int e dt + K_d \frac{de}{dt}$$

In this , e is the system output error, $\frac{de}{dt}$ is the reciprocal of the error, $\int e dt$ is the error integral, K_p is a proportionality coefficient, K_i is the integration constant, K_d is the differential coefficient. The PID controller output is the motor voltage increment

In MATLAB Simulink module design of the control system main program structure shown in Figure 3-1.

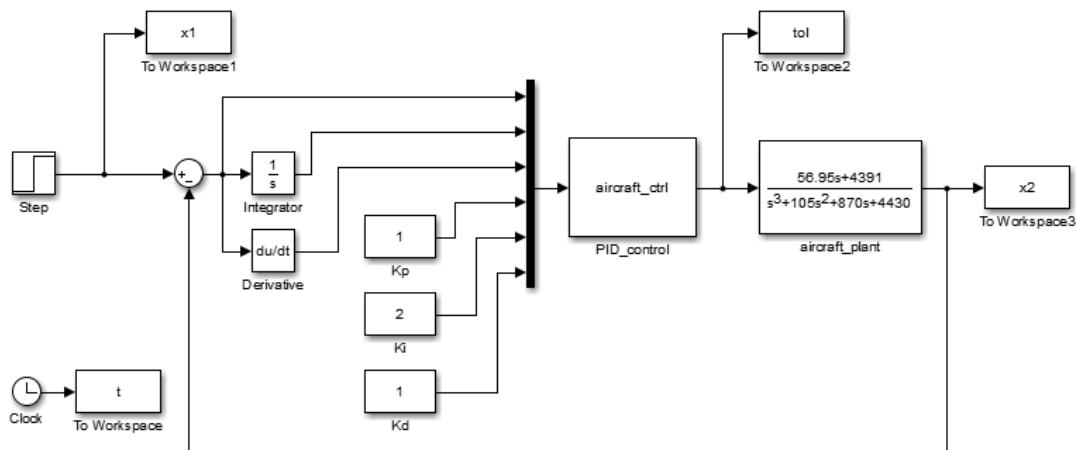


Figure 3-1 control system Simulink main program structure

The simulation results shown:

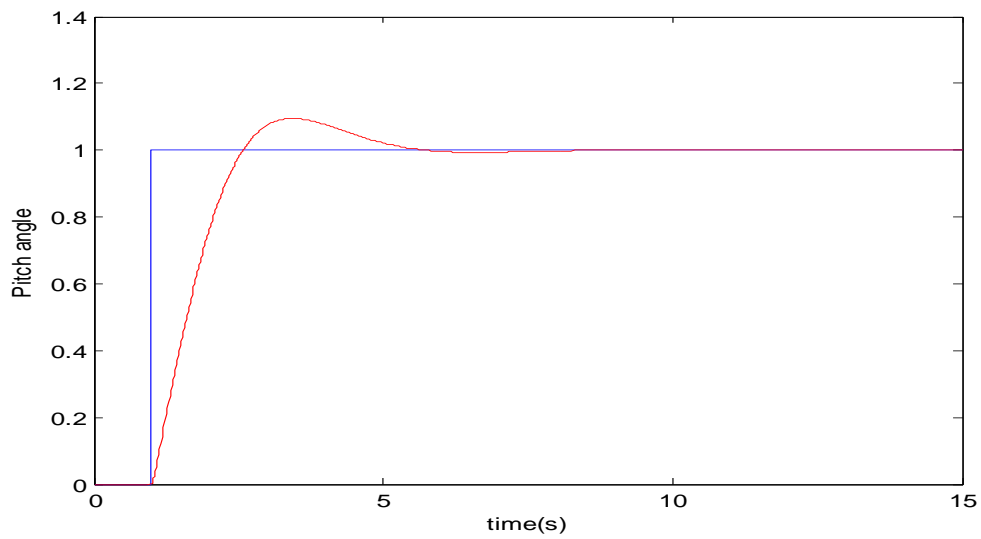


Figure 3-2 the pitch angle of the unit step response

$$G_2 = \frac{\phi}{u_2} = \frac{65s + 4560}{s^3 + 109s^2 + 1023s + 2935}$$

For the roll channel, the transfer function is:

The simulation results shown:

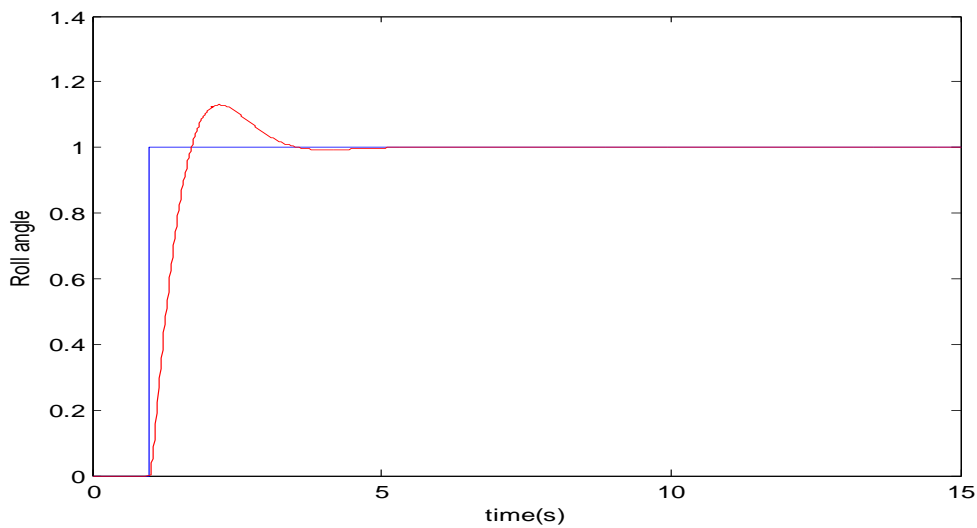


Figure 3-3 the Roll angle of the unit step response

$$G_3 = \frac{\psi}{u_3} = \frac{105}{s^2 + 413s}$$

For the yaw channel, the transfer function is:

The simulation results shown:

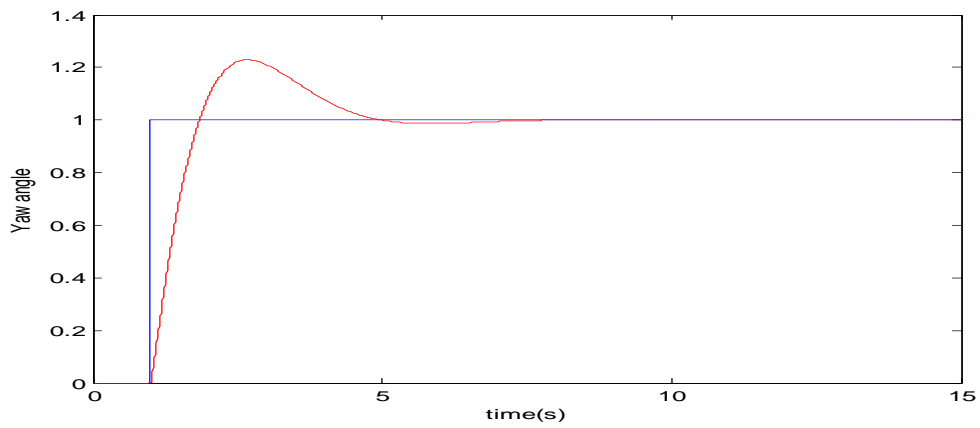


Figure 3-4 the Yaw angle of the unit step response

$$G_4 = \frac{z}{u_4} = \frac{\dot{z}}{su_4} = \frac{1.63}{s(s+5)}$$

For the Z-axis direction, the transfer function is:

The simulation results shown:

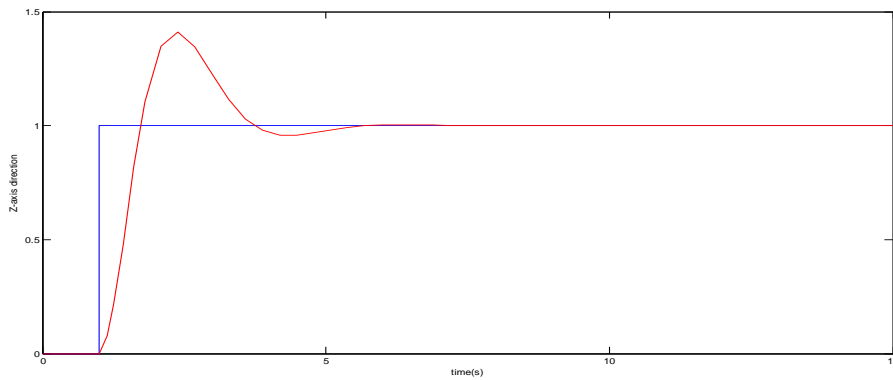


Figure 3-5 the Z-axis direction of the unit step response

In order to verify the feasibility of the PID controller. We transplanted the PID algorithm to the home-made four rotor unmanned aerial vehicle for off-field flight(in Figure 3-6), and determine the final control parameters. The data collected by these sensors is transmitted to the host computer via wifi. Test results shown in Figure 3-7



Figure 3-6 the home-made four rotor unmanned aerial vehicle

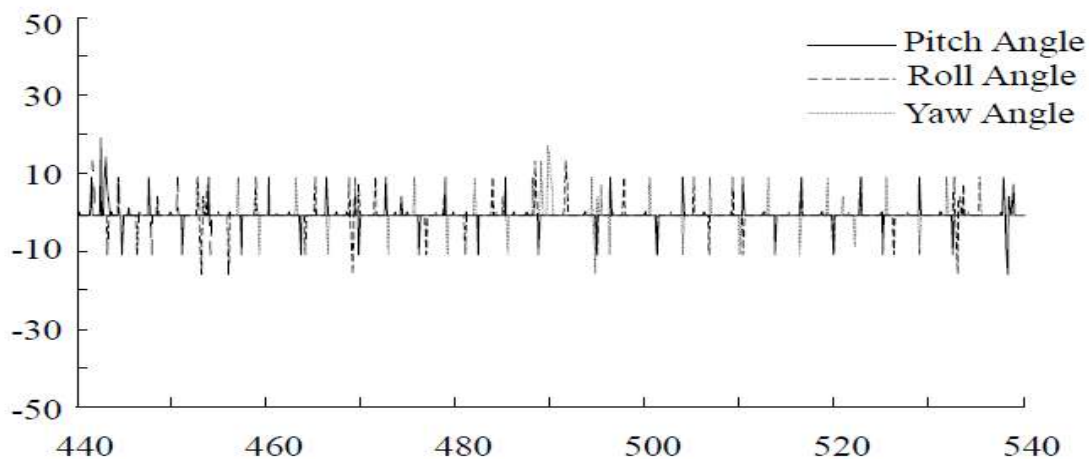


Figure 3-7 The data collected by sensors

Among them, the horizontal axis for the second, the vertical axis of the angle, the unity of the order of magnitude for the 10^{-1} Four-rotor aircraft in the 44 s off the ground, 44 ~ 54 s in the air flight, pitch angle, roll angle and heading angle of the change is small (within 5°).

V. CONCLUSION

The experimental results show that the PID control can adjust the attitude change caused by the disturbance of the breeze.

MATLAB simulation shows: The four-rotor unmanned aerial vehicle (UAV) system has a higher response speed, and the PID controller can reduce the steady-state error of the system, which makes the stability of the whole four-rotor UAV system higher, Interference is stronger, the effect of attitude control is more obvious.

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